Structures of Expectation Values of Flavor Neutrino Charges with Respect to Neutrino-Source Hadrons

Kanji Fujii and Takashi Shimomura

Department of Physics, Faculty of Sciences, Hokkaido University, Sapporo 060-0810, Japan

In the framework of quantum field theory, we examine what physical implications are included in the expectation values of the flavor neutrino charges at the time x^0 with respesct to the state $|\Psi(x^0)\rangle$, which is taken so as to coincide with the neutrino source state such as a charged pion at the time x_I^0 $(< x^0)$.

Purpse — In the quantum field theory,[1)] the expectation value of a physical observable $F(x)$ at a space-time point $x = (\mathbf{x}, x^0)$ with respect to the state $|\Psi(x^0)\rangle$ is given by, in the interaction representation,

$$\langle\Psi(x^0)|\, F(x)\, |\Psi(x^0)\rangle = \langle\Psi(x_I^0)|\, S^{-1}(x^0, x_I^0) F(x) S(x^0, x_I^0)\, |\Psi(x_I^0)\rangle, \quad (1)$$

$$S(x^0, x_I^0) = \sum_{m=0} (-i)^m \int_{x_I^0}^{x^0} d^4y_1 \int_{x_I^0}^{y_1^0} d^4y_2 \cdots \int_{x_I^0}^{y_{m-1}^0} d^4y_m H_{int}(y_1)\cdots H_{int}(y_m). \quad (2)$$

In the following we consider the expectation values of the flavor neutrino charges $N_\rho(x^0), \rho =$ the flavor suffix $(e, \mu, \cdots)$, with respect to the state $|\Psi(x^0)\rangle$ which coincides with the neutrino-source state $|\Psi(x_I^0)\rangle$ such as a charged pion at the time x_I^0 $(< x^0)$, and point out that those expectation values lead to a possible way for treating unifiedly various weak decay probabilities of neutrino-source particles as well as the neutrino oscillation, and also give a suggestion how to construct the state of flavor-neutrino with momentum $\mathbf{k}$.

Relevant quantities and relations — The total Lagrangian $\mathcal{L}(x) = \mathcal{L}_0(x) + \mathcal{L}_{int}(x)$ related to neutrinos is taken to be

$$\mathcal{L}_0(x) = -\bar{\nu}_F(x)[\partial\!\!\!/ + M]\nu_F(x), \qquad M^\dagger = M, \quad (3)$$

$$\mathcal{L}_{int}(x) = -[\bar{\nu}_F(x) J_F(x) + \bar{J}_F(x)\nu_F(x)] = -H_{int}(x); \quad (4)$$

here, $\nu_F(x)$ is a set of the flavor neutrino fields $\nu_\rho(x)$'s, and a set of the mass eigenfields $\nu_j(x)$'s is defined by

$$\nu_F(x) = Z^{1/2}\nu_M(x),\ Z^{1/2^\dagger} M Z^{1/2} = M_{diag}, \quad Z^{1/2^\dagger} Z^{1/2} = I,\ Z^{1/2} = [Z_{\rho j}^{1/2}]; \quad (5)$$

$$\nu_F(x) = \begin{bmatrix} \nu_e(x) \\ \nu_\mu(x) \\ \vdots \end{bmatrix}, \quad \nu_M(x) = \begin{bmatrix} \nu_1(x) \\ \nu_2(x) \\ \vdots \end{bmatrix}, \quad \mathrm{diag}(M_{diag}) = (m_1, m_2, \cdots)\ ; \quad (6)$$

the source function $J_F(x)$ is for simplicity assumed to include no neutrino fields, and $\mathcal{L}_{int}(x)$ is the local Fermi interaction obtained in accodance with the electro-weak

unified theory. We employ Greek and Roman indices, $(\rho, \sigma, \lambda, \cdots)$ and $(i, j, l, \cdots)$, to represent the flavor and the mass degrees of freedom, respectively.

It is noted that $F^H(x) \equiv S^{-1}(x^0, x_I^0)F(x)S(x^0, x_I^0)$ appearing in (1) is a quantity in Heisenberg representation, which is taken so as to coincide with the interaction representation at the time x_I^0. Hereafter we denote quantities in Heisenberg and the interaction representations by attaching and omitting the prefix H, respectively. We see that, due to (5), $(\not\partial + M_{diag})\nu_M(x) = 0$ leads to $(\not\partial + M)\nu_F(x) = 0$ and $(\not\partial + M)\nu_F^H(x) = -J_F^H(x)$. The integral of the latter is easily expressed in the form of so-called Yang-Feldman equation[2)] when the field-mixing exists. Its implication in the neutrino problem will be discussed elsewhere. In the following we consider directly the perturbative expansion of $S(x^0, x_I^0)$ with respect to the weak interaction

$$H_{int}(x) = \sum_\sigma \frac{G_F}{\sqrt{2}} i\bar{\nu}_\sigma(x)v^a l_\sigma(x) j_a^{had}(x)^\dagger + H.c., \tag{7}$$

where $v^a = \gamma^a(1+\gamma_5)$; G_F is Fermi coupling constant. We employ Kramers representation[3)] of γ^a-matrices, $\overrightarrow{\gamma} = -\rho_y \otimes \overrightarrow{\sigma}$, $\gamma^4 = \rho_x \otimes I$, $\gamma_5 = -\rho_z \otimes I$. The flavor neutrino charge N_ρ, the expectation value of which is to be examined, is defined by

$$N_\rho(x) \equiv : i\int d^3x j_{(\rho)}^4(\mathbf{x}, x^0) :, \tag{8}$$

where the 4-vector current of ν_ρ-field is given by $j_{(\rho)}^a(x) = -i\bar{\nu}_\rho(x)\gamma^a\nu_\rho(x)$. The symbol : : represents the normal product with respect to the momentum-helicity creation and annihilation operstors of the $\nu_M(x)$ field, which is expanded as

$$\nu_i(x) = \sum_{\mathbf{k},r} \frac{1}{\sqrt{V}} \left[\alpha_j(kr)u_j(kr)e^{i(kx)} + \beta_j^\dagger(kr)v_j(kr)e^{-i(kx)}\right]; \tag{9}$$

here $(kx) = \mathbf{k}\cdot\mathbf{x} - \omega_j(k)x^0$ with $\omega_j(k) = \sqrt{\mathbf{k}^2 + m_j^2}$; "$r$" represents the helicity, $r = \uparrow\downarrow$; $(i\not k + m_j)u_j(kr) = 0$, $(-i\not k + m_j)v_j(kr) = 0$; α_j, β_j and their Hermitian conjugates satisfy $\{\alpha_j(kr), \alpha_i^\dagger(k'r')\} = \{\beta_j(kr), \beta_i^\dagger(k'r')\} = \delta_{ji}\delta_{rr'}\delta(\mathbf{k}, \mathbf{k}')$, others $= 0$; the concrete forms of $u_j(kr)$ and $v_j(kr)$ are given in Refs.3)–5).

Expectation values of flavor neutrino charges — We investigate the structure of

$$\langle A(x_I^0)|\, S^{-1}(x^0, x_I^0)N_\rho(x^0)S(x^0, x_I^0)\,|A(x_I^0)\rangle = \langle A(x_I^0)|\, N_\rho^H(x^0)\,|A(x_I^0)\rangle, \tag{10}$$

where $|A(x_I^0)\rangle$ $(x_I^0 \le x^0)$ is a hadronic state with no neutrinos and plays a role of a neutrino source, such as one-pion state. For simplicity, we consider the case where an A-particle is a (pseudo-)scalar one with decay modes caused by (7). The contributions of G_F 0-th and first orders are easily seen to vanish. There are two kinds of G_F^2-order connected contributions, expressed as

$$\begin{aligned}&\langle A(x_I^0)|\, N_\rho^H(x^0)\,|A(x_I^0)\rangle_{con(I)} \\ &\qquad \equiv \langle A(x_I^0)| \int_{x_I^0}^{x^0} d^4z \int_{x_I^0}^{x^0} d^4y H_{int}(z)N_\rho(x^0)H_{int}(y)\,|A(x_I^0)\rangle_{con},\end{aligned} \tag{11}$$

$$
\begin{aligned}
\langle A(x_I^0)|\, N_\rho^H(x^0)\, |A(x_I^0)\rangle_{con(II)} &\equiv \langle A(x_I^0)|\, i^2 \int_{x_I^0}^{x^0} d^4y \int_{x_I^0}^{y^0} d^4z \\
&\times \left[H_{int}(z)H_{int}(y)N_\rho(x^0) + N_\rho(x^0)H_{int}(y)H_{int}(z)\right]\, |A(x_I^0)\rangle_{con}. \qquad (12)
\end{aligned}
$$

We examine dominant contributions coming from diagrammatically lower configurations in intermediate states. Those contributions come from (11) and, in the case of a positively charged A-particle, are expressed as

$$
\begin{aligned}
\langle N_\rho(x^0); A^+(x_I^0)\rangle_{con} &\equiv \int d^3x \int_{x_I^0}^{x^0} d^4z \int_{x_I^0}^{x^0} d^4y \langle A^+(x_I^0)| \\
&\times \sum_{\lambda\sigma} \bar{J}_\lambda(z)\gamma^4 \mathcal{S}_{\rho\lambda}^{(+)}(x-z)^\dagger \cdot \mathcal{S}_{\rho\sigma}^{(+)}(x-y) J_\sigma(y)\, |A^+(x_I^0)\rangle_{con}. \qquad (13)
\end{aligned}
$$

where $\langle 0|\, \nu_\rho(x)\bar{\nu}_\sigma(y)\, |0\rangle = -i\mathcal{S}_{\rho\sigma}^{(+)}(x-y)$.

The state $|A^\dagger(x_I^0)\rangle$ is now taken to be a plane-wave, i.e. $|A^\dagger(p, x_I^0)\rangle = \alpha_A^\dagger(p)\, |0\rangle e^{iE_A(p)x_I^0}$ with $p^0 = E_A(p) = \sqrt{\mathbf{p} + m_A^2}$, $[\alpha_A(p), \alpha_A^\dagger(p')] = \delta(\mathbf{p}, \mathbf{p}')$. RHS of (13) includes a hadronic part, expressed as

$$
\begin{aligned}
\langle A^+(p, x_I^0)|\, j_a^{had}(z) \cdot j_b^{had}(y)^\dagger\, |A^+(p, x_I^0)\rangle &= \langle A^+(p, x_I^0)|\, j_a^{had}(z) \Big\{ |0\rangle\langle 0| \\
+ \sum |\text{1-particle}\rangle\langle\text{1-particle}| &+ \text{higher configurations} \Big\} j_b^{had}(y)^\dagger\, |A^+(p, x_I^0)\rangle. \qquad (14)
\end{aligned}
$$

The intermediate vacuum contribution is written as

$$
\begin{aligned}
\langle N_\rho(x^0); A(x_I^0)\rangle_{con}^{vac} &= \left[i\frac{G_F}{\sqrt{2}} f_A\right]^2 \int_{x_I^0}^{x^0} dz^0 \int_{x_I^0}^{x^0} dy^0 \frac{1}{2E_A(p)V} \sum_{\mathbf{q}} \sum_{\mathbf{k}} \delta(\mathbf{k}+\mathbf{q}, \mathbf{p}) \\
&\times \sum_{j,i,\sigma} Z_{\sigma j}^{1/2} Z_{\rho j}^{1/2*} Z_{\rho i}^{1/2} Z_{\sigma i}^{1/2*} \sum_{s,r} \bar{v}_{l\sigma}(qs)\, \not{p}(1+\gamma_5) u_j(kr)\rho_{ij}(k) \cdot \bar{u}_i(kr)\, \not{p}(1+\gamma_5) v_{l\sigma}(qs) \\
&\times e^{i(\omega_j-\omega_i)x^0} e^{i(-\omega_j-E_\sigma+E_A)z^0} e^{i(\omega_i+E_\sigma-E_A)y^0}; \qquad (15)
\end{aligned}
$$

here, f_A is the decay constant of A^+ , defined by

$$
\langle 0|\, j_a^{had}(x)^+\, |A^+(p, x_I^0)\rangle = \frac{1}{\sqrt{2E_A(p)V}} ip_a f_A e^{i(px)} \exp(iE_A(p)x_I^0); \qquad (16)
$$

$v_{l\sigma}(qs)$ is the momentum-helicity eigenfunction for $\bar{l}_\sigma$, satisfying $(-i\not{q}+m_\sigma)v_{l\sigma}(qs) = 0$, $q^0 = E_\sigma(q) = \sqrt{\mathbf{q}^2 + m_\sigma^2}$; $u_j^\dagger(kr)u_i(ks) = \rho_{ji}(k)\delta_{rs}$, $\rho_{ji}(k) = \cos[(\chi_j - \chi_i)/2]$ with $\cot\chi_j = |\mathbf{k}|/m_j$.[4),5)] Eq.(15) can be thought to lead to an oscillation formula. Some simplification of (15) is obtained if we multiply by hand to RHS of (15) a damping factor due to the decay width $\Gamma_A(p)$ of A-particle, and perform the z^0- and y^0-integrations under the condition $t\Gamma_A(p) \gg 1$, $t \equiv x^0 - x_I^0$. Although the obtained expression includes the simple t-dependent factor $exp[i(\omega_j(k) - \omega_i(k))t]$, we have to perform further the $\mathbf{k}$-integration; thus the oscillation behavior is different from that

of the well-known standard formula[6),7)] as well as of the expectation value of the flavor neutrino charge with respect to a flavor neutrino state.[8)] Detailed analyses in the plane- wave and the wave-packet $\left|A(x_I^0)\right\rangle$ cases will be done in a subsequent paper. (Some considerations on the latter case have been done in Ref.9).)

Decay probabilities of neutrino source particles — We examine what results are obtained from $\left\langle N_\rho(x^0); A^+(p, x_I^0)\right\rangle$ for sufficiently large $T = x^0 - x_I^0$. We obtain from (15) $\left\langle N_\rho(x^0); A^+(x_I^0)\right\rangle_{con}^{vac}/T$

$$\begin{aligned} &\longrightarrow \sum_{j,\sigma} |Z_{\rho j}^{1/2}|^2 |Z_{\sigma j}^{1/2}|^2 \frac{1}{(2\pi)^6} \int d^3k \int d^3q \frac{(2\pi)^4 \delta^4(k+q-p)}{2E_A(p)} \left[\frac{G_F}{\sqrt{2}} f_A\right]^2 \\ &\quad \times \sum_{s,r} i\bar{v}_{l\sigma}(qs) \not{p}(1+\gamma_5) u_j(kr) \cdot i\bar{u}_i(kr) \not{p}(1+\gamma_5) v_{l\sigma}(qs) \\ &\quad = \sum_{j,\sigma}{}' |Z_{\rho j}^{1/2}|^2 |Z_{\sigma j}^{1/2}|^2 P(A^+(p) \to \bar{l}_\sigma \nu_j); \end{aligned} \tag{17}$$

here, $P(A^+(p) \to \bar{l}_\sigma \nu_j) = P(A^+(\mathbf{p}=0) \to \bar{l}_\sigma \nu_j) \cdot (m_A/E_A(p))$; the sum $\sum_{j,\sigma}'$ is performed over j's and s's which are allowed under 4-momentum conservation; the explicit form of the decay probability $P(A^+(\mathbf{p}=0) \to \bar{l}_\sigma \nu_j)$ calculated in the lowest order of the weak interaction (7) is found e.g. in Refs. 5) and 10).

By taking into account various intermediate states as shown in (14), we can generalize (17) to each weak semileptonic decay, $A^+ \to \bar{l}_\sigma \nu_j + \text{hadron(s)}$; then, by summing over all modes, we obtain in order

$$\begin{aligned} &\left[\left\langle N_\rho(x^0); A(x_I^0)\right\rangle_{con}/T\right]_{T=x^0-x_I^0\to\infty} \quad (\equiv \left\langle N_\rho; A^+(p), \infty\right\rangle\,) = \sum_{j,\sigma}{}' |Z_{\rho j}^{1/2}|^2 |Z_{\sigma j}^{1/2}|^2 \\ &\times \left[P(A^+(p) \to \bar{l}_\sigma \nu_j) + \sum_{\text{modes}} P(A^+(p) \to \bar{l}_\sigma \nu_j + \text{hadron(s)})\right], \end{aligned} \tag{18}$$

$$\begin{aligned} &\sum_\rho \left\langle N_\rho; A^+(p), \infty\right\rangle \\ &= \sum_{j,\sigma}{}' \left|Z_{\sigma j}^{1/2}\right|^2 \left[P(A^+(p) \to \bar{l}_\sigma \nu_j) + \sum_{\text{modes}} P(A^+(p) \to \bar{l}_\sigma \nu_j + \text{hadron(s)})\right]. \end{aligned} \tag{19}$$

Judging from the meaning of the expectation value now considering, LHS of (19) may be regarded as the total leptonic decay probability of A^+-particle accompanying a neutrino; in the case of π^+, its life-time $\tau(\pi^+(p))$ is given by

$$\tau(\pi^+(p))^{-1} \simeq \sum_{j,\sigma}{}' \left|Z_{\sigma j}^{1/2}\right|^2 P(\pi^+(p) \to \bar{l}_\sigma \nu_j) \simeq \sum_j{}' \left|Z_{\mu j}^{1/2}\right|^2 P(\pi^+(p) \to \mu^+ \nu_j). \tag{20}$$

It seems worthwhile to note the relation in the case of the expectation value of the number of charged lepton l_ρ. We obtain

$$\left[\left\langle A^+(p, x_I^0)\right| N_{l\rho}^H(x^0) \left|A^+(p, x_I^0)\right\rangle_{con}/T\right]_{T\to\infty} = \sum_j{}' |Z_{\rho j}^{1/2}|^2$$

$$\times \left[P(A^+(p) \to \bar{l}_\rho \nu_j) + \sum_{\text{modes}} P(A^+(p) \to \bar{l}_\rho \nu_j + \text{hadron(s)}) \right] \tag{21}$$

for energetically allowed ρ's. We see (21) leads also to (20).

Lastly we give a remark on the relations derived by assuming the 3 flavor number and $P(\pi^+(p) \to \mu^+ \nu_j)$'s to be almost independent of all three j's as well as by noting $m_\tau \simeq 1780$ MeV. Then, from (18) we obtain

$$\begin{aligned} &\langle N_{\nu_e}; \pi^+(p), \infty \rangle \ : \ \langle N_{\nu_\mu}; \pi^+(p), \infty \rangle \ : \ \langle N_{\nu_\tau}; \pi^+(p), \infty \rangle \\ &\simeq \sum_j |Z_{ej}^{1/2}|^2 |Z_{\mu j}^{1/2}|^2 \ : \ \sum_j |Z_{\mu j}^{1/2}|^4 \ : \ \sum_j |Z_{\tau j}^{1/2}|^2 |Z_{\mu j}^{1/2}|^2. \end{aligned} \tag{22}$$

Similarly in the case of K^+, under the j-independence of $P\big(K^+ \to \bar{l}_\sigma \nu_j (+ \text{ hadron(s)})\big)$ for $\sigma = e, \mu$, we obtain

$$\begin{aligned} &\langle N_{\nu_e}; K^+(p), \infty \rangle \ : \ \langle N_{\nu_\mu}; K^+(p), \infty \rangle \ : \ \langle N_{\nu_\tau}; K^+(p), \infty \rangle \\ &\simeq \sum_j |Z_{ej}^{1/2}|^2 R_j \ : \ \sum_j |Z_{\mu j}^{1/2}|^2 R_j \ : \ \sum_j |Z_{\tau j}^{1/2}|^2 R_j \end{aligned} \tag{23}$$

with $R_j \equiv |Z_{ej}^{1/2}|^2 r(K^+ \to e^+) + |Z_{\mu j}^{1/2}|^2 r(K^+ \to \mu^+)$; here, $r(K^+ \to \bar{l}_\sigma)$ means the branching ratio of K^+ decay mode with one $\bar{l}_\sigma$. If it is allowed for us to drop $r(K^+ \to e^+)$ owing to the experimental value $r(K^+ \to e^+)/r(K^+ \to \mu^+) \simeq 4.87/66.70 \simeq 1/13.7$, we see RHS of (23) is nearly equal to that of (22).

As to the neutron life time, we also obtain

$$\left[\frac{1}{2} \sum_r \langle N_\rho(x^0); n(p, x_I^0) \rangle_{con} / T \right]_{T \to \infty} = \sum_j{}' |Z_{\rho j}^{1/2}|^2 |Z_{ej}^{1/2}|^2 P(n(\mathbf{p}) \to p e^- \bar{\nu}_j). \tag{24}$$

Assuming the independence of $P(n \to p\ e\ \bar{\nu}_j)$ on j's, we obtain

$$\begin{aligned} &\langle N_{\nu_e}; n(p), \infty \rangle \ : \ \langle N_{\nu_\mu}; n(p), \infty \rangle \ : \ \langle N_{\nu_\tau}; n(p), \infty \rangle \\ &\simeq \sum_j |Z_{ej}^{1/2}|^4 \ : \ \sum_j |Z_{\mu j}^{1/2}|^2 |Z_{ej}^{1/2}|^2 \ : \ \sum_j |Z_{\tau j}^{1/2}|^2 |Z_{ej}^{1/2}|^2. \end{aligned} \tag{25}$$

In the standard theory of the neutrino oscillation in vacuum,[6)] the transition probability for $\nu_\sigma \to \nu_\rho$ is given by

$$P_{\nu_\sigma \to \nu_\rho}(t = x^0 - x_I^0) = |\sum_j U_{\sigma j} e^{i\omega_j(\mathbf{k})t} U_{\rho j}^*|^2, \quad U^\dagger U = I. \tag{26}$$

The time average of $P_{\nu_\sigma \to \nu_\rho}(t)$[6),7)] over a sufficiently long time interval $T \gg 4\pi k/|\Delta m_{ji}^2|$ for any pair (j, i), $j \neq i$ is given by

$$\langle P_{\nu_\sigma \to \nu_\rho} \rangle_T \equiv \int_0^T dt P_{\nu_\sigma \to \nu_\rho}(t)/T = \sum_j |U_{\rho j}|^2 |U_{\sigma j}|^2, \tag{27}$$

which leads to the same ratio relations as (22) and (25) if we set $U = Z^{1/2}$. Although this equality is taken in the standard theory,[6),7)] there is some problem for understanding this equality on the basis of the quantum field theory, as first stressed by Blasone and Vitiello; a related remark is to be given below 4), 5), 11). From our viewpoint, the relation (27) should be understood as the relations (22) and (25) which have a field theoretical basis.

Final remarks and conclusion — It seems meaningful for us to consider whether or not we can construct such one ν_ρ-state (with momentum-helicity $(\mathbf{k}, r)$) that the absolute square of the transition matrix element of $A^+ \to \bar{l}_\sigma \nu_\rho$ leads to Eq.(15).

We define

$$\left|\Psi_{\nu_\rho}(\mathbf{k}r; x^0)^d\right\rangle \equiv \frac{1}{\sqrt{V}} \int d^3x e^{i\mathbf{k}\cdot\mathbf{x}} (\nu_\rho(x) P(r))^d \,|0\rangle, \tag{28}$$

where $P(r)$ is the helicity projection matrix[3)−;5)] "d" specifies the 4-spinor component. RHS of (28) is expressed as $\sum_j Z^{1/2*}_{\rho j} \alpha_j^\dagger(kr; x^0) u_j^\dagger(kr)^d \,|0\rangle$ with $\alpha_j^\dagger(kr; x^0) = \alpha_j^\dagger(kr) \exp(i\omega_j(k)x^0)$; thus, we have $\sum_d \left\langle \Psi_{\nu_\sigma}(\mathbf{k}r; x^0)^d \middle| \Psi_{\nu_\rho}(\mathbf{k}'s; x^0)^d \right\rangle = \delta_{\sigma\rho}\delta(\mathbf{k}, \mathbf{k}')\delta_{rs}$. Let us consider the transition matrix element

$$\begin{aligned} M(A^\dagger(p) \to \bar{l}_\sigma(qs)\nu_\rho(kr); x^0, x_i^0)^d \equiv \Big\langle \Psi_{\nu_\rho}(\mathbf{k}r; x^0)^d \Big| \,\beta_{l_\sigma}(qs; x^0) \\ \times (-i) \int_{x_I^0}^{x^0} d^4z H_{\rm int}(z) \alpha_A^\dagger(p; x^0) \,|0\rangle, \end{aligned} \tag{29}$$

where $\beta_{l_\sigma}(qs; x^0) = \beta_{l_\sigma}(qs) \exp(iE_\sigma(q)x^0)$, $\alpha_A(p; x^0) = \alpha_A(p) \exp(iE_A(p)x^0)$. As easily confirmed, we obtain

$$\sum_{\mathbf{q},\mathbf{k}} \sum_{\sigma,d,r,s} \left| M(A^\dagger(p) \to \bar{l}_\sigma(qs)\nu_\rho(kr); x^0, x_i^0)^d \right| = \text{RHS of (15)}. \tag{30}$$

This relation suggests that the state (28) is an appropriate one ν_ρ state (with $(\mathbf{k}, r)$) in quantum field theory. If so, we have

$$\sum_d \left\langle \Psi_{\nu_\rho}(\mathbf{k}r; x^0)^d \middle| \Psi_{\nu_\sigma}(\mathbf{k}r; x_I^0)^d \right\rangle = \sum_j Z_{\rho j}^{1/2} e^{-i\omega_j(x^0 - x_I^0)} Z_{\sigma j}^{1/2^*}, \tag{31}$$

leading to (26) with $U = Z^{1/2}$.

A simple way for obtaining the oscillation in space is to substitute L (the distance) for t and to employ the wave-packets of the relevant particles. Along this line, it is necessary for us to make clear what differences in the oscillation behavior exist among the present approach, the standard formula and the approach of intermediate virtual neutrinos.[12),13)]

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